# B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

## Subject : Mathematics

## Course : BMH5DSE11

## (Linear Programming)

**Time: 3 Hours** **Full Marks: 60**

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*Notation and symbols have their usual meaning.*

**1.** Answer *any ten* questions: 2×10=20

(a) Define basic feasible solution of an LPP.

(b) What do you mean by convex hull and convex polyhedron? 1+1

(c) Is the following set $A$ in $R^2$ is convex? Justify with your reason.
$A = \{(x, y): x > 0, y > 0 \text{ and } xy \leq 1\}$

(d) Find the extreme points of the set $A = \{(x, y): |x| \leq 2, |y| \leq 2\}$.

(e) What do you mean by alternative optima of an LPP?

(f) Find a basic feasible solution of the following system of equations:

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

(g) Test whether the following set of vectors are linearly dependent or not.
$\{(3,0,2), (7,0,9), (4,1,2)\}$

(h) Find the condition under which the following game problem will be a fair game.
$\begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$ where $a, b, c, d$ are all $\geq 0$.

(i) Determine the value of θ so that the game with following payoff matrix is strictly determinable.

| | Player B | | |
|---|---|---|---|
| Player A | θ | 6 | 2 |
| | − 1 | θ | − 7 |
| | −2 | 4 | θ |

(j) Give an example of symmetric game and find its value.

(k) Prove that the solution of a transportation problem with 2 origins and 3 destinations is bounded. 1+1

(l) Find the dual of the primal problem given by

Minimize $Z = -6x_1 - 8x_2 + 10x_3$

subject to

$x_1 + x_2 - x_3 \geq 2,$

$2x_1 - x_3 \geq 1,$

$x_1, x_2, x_3 \geq 0.$

(m) State complementary slackness theorem.

(n) "All boundary points are not necessarily extreme points."— Justify this statement with example.

(o) Prove that if a linear programming problem has two feasible solutions, then it has an infinite number of feasible solution.

**2.** Answer *any four* questions: 5×4=20

(a) Use Simplex method to obtain inverse of the matrix $\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$.

(b) Solve the following linear programming problem:

Maximize $Z = 3x_1 + 4x_2$

Subject to $x_1 - x_2 \geq 0,$

$-x_1 + 3x_2 \leq 3$

and $x_1, x_2 \geq 0,$

(c) Use Dual simplex method to solve the LPP:

Minimize $Z = 10x_1 + 6x_2 + 2x_3$

subject to $-x_1 + x_2 + x_3 \geq 1,$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0.$

(d) Use dominance property to reduce the following payoff matrix to $2 \times 2$ matrix and hence solve the problem:

Player A

| | | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| | $B_1$ | 4 | 2 | 0 | 2 | 1 | 1 |
| | $B_2$ | 4 | 3 | 1 | 3 | 2 | 2 |
| Player B | $B_3$ | 4 | 3 | 7 | −5 | 1 | 2 |
| | $B_4$ | 4 | 3 | 4 | −1 | 2 | 2 |
| | $B_5$ | 4 | 3 | 3 | −2 | 2 | 2 |

(e) Prove that any points of a convex polyhedorn can be expressed as a convex combination of its extreme points.

(f) Prove that the number of basic variables in a transportation problem with 2 origins and 3 destinations is at most 4.

**3.** Answer *any two* questions: 10×2=20

(a) (i) Use Vogel's Approximation Method to find the initial B.F.S. of the following transportation problem:

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $a_i$ |
|---|---|---|---|---|---|
| $O_1$ | 1 | 2 | 1 | 4 | 30 |
| $O_2$ | 3 | 3 | 2 | 1 | 50 |
| $O_3$ | 4 | 2 | 5 | 9 | 20 |
| $b_j$ | 20 | 40 | 30 | 10 | |

(ii) Solve graphically the game whose payoff matrix is given below: 5+5

| | Player B | | |
|---|---|---|---|
| | | $B_1$ | $B_2$ |
| Player A | $A_1$ | 2 | 7 |
| | $A_2$ | 3 | 5 |
| | $A_3$ | 11 | 2 |

(b) (i) Prove that the set of optimal strategies for each player in an $m \times n$ matrix game is a convex set.

(ii) Solve the travelling salesman problem: 5+5

| | | To | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | E |
| From | A | ∞ | 6 | 12 | 6 | 4 |
| | B | 6 | ∞ | 10 | 5 | 4 |
| | C | 8 | 7 | ∞ | 11 | 3 |
| | D | 5 | 4 | 11 | ∞ | 5 |
| | E | 5 | 2 | 7 | 8 | ∞ |

(c) (i) Find the maximum value of $Z = 6x + 8y$.

subject to $5x + 2y \leq 20$

$x + 2y \geq 10$

$x, y \geq 0$

by solving its dual problem.

(ii) Solve the following assignment problem: 5+5

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 62 | 78 | 50 | 101 | 82 |
| 2 | 71 | 84 | 61 | 73 | 59 |
| 3 | 87 | 92 | 111 | 71 | 81 |
| 4 | 48 | 64 | 87 | 77 | 80 |

(d) (i) Solve the following LPP by two phase method:

Maximize $z = 3x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_1 \leq 4$

and $x_1, x_2 \geq 0$

(ii) Is

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $O_1$ | | | 50 | 20 |
| $O_2$ | 55 | | | |
| $O_3$ | 30 | 35 | | 25 |

an optimal solution of the following transportation problem? 5+5

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $a_i$ |
|---|---|---|---|---|---|
| $O_1$ | 6 | 1 | 9 | 3 | 70 |
| $O_2$ | 11 | 5 | 2 | 8 | 55 |
| $O_3$ | 10 | 12 | 4 | 7 | 90 |
| $b_j$ | 85 | 35 | 50 | 45 | |

If not, modify it to obtain a better feasible solution.

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# B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

## Subject : Mathematics

## Course : BMH5DSE12

## (Number Theory)

**Time: 3 Hours** **Full Marks: 60**

*The figures in the margin indicate full marks.*
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**1.** Answer *any ten* questions: 2×10=20

(a) Show that Goldbach conjecture implies that every even integer greater than 5 is a sum of three primes.

(b) For any integers $a, b, c$ prove that $a|b$ and $b|a$ iff $a = \pm b$.

(c) Prove that $(n^2 + 2)$ is not divisible by 4 for any integer $n$.

(d) Find the remainder when $3^{100}$ is divided by 5.

(e) State Fermat's Little Theorem.

(f) Show that $19^{20} \equiv 1 \pmod{181}$.

(g) Prove that if $8 \times 7 \equiv 2 \times 7 \pmod 6$ and $(7, 6) = 1$, then $8 \equiv 2 \pmod 6$.

(h) Solve $x^2 + 3x + 11 \equiv 0 \pmod{13}$.

(i) If $p$ is prime, prove that $2(p-3)! + 1 \equiv 0 \pmod p$.

(j) Find the missing digit in the number 287*932 if it is divisible by 13.

(k) Prove that $2^n < n!$ for $n \in \mathbb{N}$ and $n \geq 4$.

(l) If $d_1, d_2, \ldots, d_r$ be the list of all positive divisors of a positive integer $n$, prove that $\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_r} = \frac{\sigma(n)}{n}$.

(m) Solve the linear congruence: $28x \equiv 63 \pmod{105}$.

(n) If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_r^{\alpha_r}$ where $p_1, p_2, \ldots, p_r$ are prime to one another, find $\phi(n)$ ($\alpha_1, \alpha_2, \ldots, \alpha_r$ are positive integers).

(o) Prove that $2^n - 1$ has at least $n$ distinct prime factors.

**2.** Answer *any four* questions: 5×4=20

(a) (i) Find $\sigma(360)$ and $\sigma(900)$.

(ii) Let $k > 1$ and $2^k - 1$ is a prime. If $n = 2^{k-1}(2^k - 1)$, then show that $n$ is a perfect number. 2+3

(b) Prove that Möbius μ-function is a multiplicative function.

(c) State and prove Euclid's Theorem.

(d) Prove that $an \equiv bn \pmod{m}$ if and only if $a \equiv b \left(\bmod \frac{m}{(m,n)}\right)$, where $a, b, m, n$ are integers.

(e) Show that $3^{4n+2} + 5^{2n+1} \equiv 0 \pmod{14}$.

(f) Find the primitive roots of 41.

**3.** Answer *any two* questions: 10×2=20

(a) (i) Prove that every integer $(n > 1)$ can be expressed as a product of finite number of primes.

(ii) Find the remainder when $2^{73} + 14^3$ is divided by 11. 8+2

(b) (i) Find the digit in unit place of $3^{400}$.

(ii) State and prove Chinese Remainder Theorem. 2+8

(c) (i) Prove that $7 | (2222^{5555} + 5555^{2222})$.

(ii) Solve the linear Diophantine equation: $221x + 35y = 11$ 5+5

(d) (i) Find the least natural number which when divided by 7, 10 and 11 leaves in order the remainders 1, 6 and 2.

(ii) Let $p$ be an odd prime. Then prove that the congruence $x^2 \equiv -1 \pmod{p}$ has a solution if and only if $p \equiv 1 \pmod{4}$. 5+5

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# B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

## Subject : Mathematics

## Course : BMH5DSE13

## (Point Set Topology)

**Time: 3 Hours** **Full Marks: 60**

*The figures in the margin indicate full marks.*
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*Notation and symbols have their usual meaning.*

**1.** Answer *any ten* questions: 2×10=20

(a) State the axiom of choice.

(b) Define product topology.

(c) Define addition and multiplication of two cardinal numbers. 1+1

(d) If $u, v$ and $w$ are cardinal numbers then prove that $u(vw) = (uv)w$.

(e) Let $(A, \leq)$ be a totally ordered set and $x \leq y,\ x, y \in A$. Prove that $A_x \subset A_y$, where $A_x$ and $A_y$ denote the initial segments determined by $x$ and $y$ respectively.

(f) Let $(X, \tau)$ be a topological space. Prove that a subset $G$ of $X$ is open if and only if it is a neighbourhood of each of its points.

(g) Let $(X, \tau), (Y, \tau')$ and $(Z, \tau'')$ be three topological spaces. Let $f: (X, \tau) \to (Y, \tau')$ and $g: (Y, \tau') \to (Z, \tau'')$ be two continuous functions. Prove that $gof: (X, \tau) \to (Z, \tau'')$ is continuous.

(h) Define a cofinite topological space.

(i) Prove that a topological space $(X, \tau)$ is connected if and only if it has no non-empty proper subset which is both open and closed.

(j) Let $A$ and $B$ be two connected subsets in a topological space $(X, \tau)$ with $A \cap B \neq \emptyset$. Prove that $A \cup B$ is connected in $(X, \tau)$.

(k) Let $Q$ be the set of rational numbers equipped with subspace topology of usual topology of $\mathbb{R}$. Examine if $Q$ is connected.

(l) Prove that each cofinite space is compact.

(m) Let $X$ be a compact space and $Y$ be a $T_2$ space and $f : X \to Y$ be continuous. Prove that $f$ is a closed map. 2

(n) Show that a circle or a line or a parabola in $\mathbb{R}^2$ is not homeomophic to a hyperbola. 2

(o) Using the definition of a compact set, prove that the open interval (0, 1) is not a compact subset of $\mathbb{R}$, the real number space equipped with usual topology. 2

**2.** Answer *any four* questions: 5×4=20

(a) Prove that a sequentially compact metric space is compact.

(b) Let $f:(X,\tau)\rightarrow(Y,\tau')$ be a function. When is $f$ called closed? Prove that f is closed map if and only if $f(\overline{A})\supset\overline{f(A)}$ for every $A\subset X$. 1+2+2

(c) If $u,v,w$ are cardinal numbers show that $(uv)^w = u^w\cdot v^w$.

(d) Let $(A,\leq)$ be a well ordered set. Prove that

(i) $A$ is order isomorphic to no initial segment of $A$,

(ii) if $A_x\cong A_y$, then $x=y$. 3+2

(e) Define a path connected space. Prove that every path connected space is connected. 1+4

(f) Prove that a topological space is locally connected if and only if each component of an open set is open. 2½+2½

**3.** Answer *any two* questions: 10×2=20

(a) (i) Prove that for any two cardinal numbers $u$ and $v$, either $u\leq v$ or $v\leq u$.

(ii) If $u, v$ and $w$ are cardinal numbers, then prove that $u^v u^w = u^{v+w}$. 5+5

(b) (i) Prove that a compact subset in a metric space is closed and bounded.

(ii) When is a topological space said to be locally connected? Is every connected space locally connected? Support your answer. (2+3)+(1+4)

(c) (i) Let $(X,\tau)$ and $(Y,\tau')$ be two topological spaces and $f:X\rightarrow Y$ be a mapping. Let $\{x_n\}$ be a sequence in $X$ converging to $x$. If $f$ is continuous then show that the sequence $\{f(x_n)\}$ converges to $f(x)$ in $Y$. Does the converse hold? Support your answer.

(ii) Let $C$ be a connected subset of a topological space $(X,\tau)$. Show that $\bar{C}$ is connected. Hence or otherwise show that component in a topological space is closed. (2+4)+(3+1)

(d) (i) Define a locally compact space. Prove that a closed subset of a locally compact space is locally compact.

(ii) If $u,v,w$ are cardinal numbers, prove that $(u^v)^w = u^{vw}$. (1+4)+5

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